PERGAMON

International Journal of Solids and Structures 37 (2000) 93–104

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Mechanics of cellular and other low-density materials

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Abstract

Both two-dimensional and three-dimensional low density materials are surveyed. The microstructure is usually in a cellular form with some characteristic dimension(s) being small compared to the cell size and with the density range approaching that at which the loss of material integrity occurs. The mechanical properties of stiffness and strength are considered, along with applications and future opportunities. Published by Elsevier Science Ltd.

1. Introduction

This survey is concerned with the mechanical properties for low density solid type materials. Typically for such materials one wants to meet some requirement for a reduced level of density but also there is needed to be some reasonable level of mechanical integrity, meaning the ability to bear load. These two requirements often come into conflict, or at least involve a 'trade-off' characteristic between them. To illustrate this, one need look no further than the classical formula for the effective modulus of an elastic material containing a dilute suspension of spherical voids. This formula is

$$\frac{E}{E_m} = 1 - \frac{3(1-\nu_m)(9+5\nu_m)}{2(7-5\nu_m)}c,$$

where E_m and ν_m are the basic material modulus and Poisson's ratio and $c \ll 1$ is the volume fraction of the voids. At a value of $\nu_m = 1/3$ the above formula is

$$\frac{E}{E_m} = 1 - 2c.$$

It is seen that the effective modulus, E, is degraded at a faster rate, than that at which material volume and overall density are diminished, which go as $(1-c)$. The corresponding two dimensional case

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0020-7683/00/$ - see front matter. Published by Elsevier Science Ltd.
PII: S0020-7683(99)00080-3

gives an even stronger illustration of this effect. Nevertheless, for a variety of applications it is still vitally important to achieve a certain level of (low) density. The challenge then becomes one of introducing the porosity in such a manner that the mechanical behavior is optimized for that level of density. The microstructure of the porosity becomes the overwhelmingly determinate feature that controls mechanical behavior. Much of the current and future research intimately relates to understanding and controlling the morphological microstructure of the low density material in order to achieve certain levels of performance.

The applications for low density materials are far ranging from consumer products such as food containers, to packaging materials, to crash protection energy absorbers. The economic level of the products spreads across the spectrum from larger volume home products to high technology forms such as high performance honeycomb in aerospace applications and in sporting goods. There is in fact a driving force to re-examine nearly all products at all levels asking the question if they can be made lighter in a cost effective manner, and still perform the intended function.

The types of materials to be considered here are commonly called cellular materials, although the term is not inclusive enough to encompass all low density forms. This survey of the status is limited to the behavior of solids, not fluids, and it is concerned only with mechanical properties and behavior. Lower order properties such as conductivity are much more easily treated than is the mechanical case.

There are many accessible references to the general subject. The most widely known and used reference is that of Gibson and Ashby (1997). These authors, through the body of literature on cellular solids which they have produced, could be called the pioneers of the modern era in this field. Historically, the interest in ultra porous materials arose through the fluid side in general and through liquid induced foams in particular. Cellular forms dominated by surface tension effects have been of interest and even fascination for a very long time going back to the formulation of the basic rules for geometric stability given by Plateau (1873). The modern side has had developments progressing in unison in both the fluids and the solids sides. The fluidic aspects of the subject include the behaviors of liquid foams, types of emulsions, suspensions and a variety of related forms. The solid type applications have already been mentioned. An excellent review of both the fluids and the solids forms entitled "*Foam Micromechanics*" has been given by Kraynik et al. (1998). The present references should be supplemented with those from that source. Also, Kraynik and Warren (1994) have given a review of mechanical behavior for cellular solids.

The present review is concerned with mechanical behavior near the limit of maximum porosity, beyond which the material loses mechanical integrity. As such, the scope is more specialized than that for all porous materials over the full range of possible porosity. With behavior near the extreme limit of the possible porosity, mechanics types solutions for properties often represent the leading terms in an expansion of a type to be shown. Many areas of specific and self contained application will only be briefly mentioned, since many of the examples warrant surveys by themselves.

The reasonable place to begin this survey is with the topic of minimal surfaces. Cellular forms admit a geometric characterization in terms of the surface content, or surface area per unit volume. Opening the subject in this manner will give the first discriminate of different cellular forms which ultimately will be very useful.

2. Minimal surfaces

The interest in minimal surfaces has existed for a very long time and has been deeply examined, mostly by mathematicians and physicists. The basic question is this: what is the repeating cellular form that subdivides space into cells, with the cells having a minimal surface area. This classical topic has been advanced into the modern developments and has become of great relevance in understanding the

Table 1
3-D surface properties. Planar faces (warped faces)

Cell	Faces (Average)	$\frac{A}{V^{2/3}}$
Tetrahedra + Octahedra	5-1/3	6.234
Cube	6	6
Rhombic Dodecahedron	12	5.345
Weaire–Phelan	13-1/2	5.310 (5.290)
Kelvin	14	5.315 (5.306)

mechanical behavior of cellular forms. Examining minimal surfaces begins the connection with the effect of microstructure variation and this approach invests a high degree of order and logic into the proceedings.

Lord Kelvin (1887) proposed a minimal surface cellular form and it stood uncontested for over 100 years. The proposed Kelvin cell is that of a truncated octahedron, the tetrakeidecahedron, containing 14 faces, 8 hexagons and 6 squares. The form fills 3-space and repeats in a periodic manner. In a recent, and much recognized development, Weaire and Phelan (1994) have identified a form with a lower surface content than that of the Kelvin cell. This form contains combinations of two cell types, a 14 sided form with faces of pentagons and hexagons combined with the 12 sided regular dodecahedron. The average number of sides for the Weaire–Phelan form is 13-1/2.

A nondimensional measure is needed to compare the surface content for different cell types. A convenient measure is given by

$$\zeta_{3D} = \frac{A}{V^{2/3}},$$

where A is the cell area and V its volume. When two or more cell types are involved, as with the Weaire–Phelan form, the value of ζ is found by averaging on $\zeta^{3/2}$ with volume weighting for the different cell types.

Table 1 gives the surface measure for different cellular forms that fill space. All cases have cubic symmetry, and this is the highest order symmetry that can be obtained by a space filling periodic repeating cell pattern. The close proximity of the Kelvin cell and the Weaire–Phelan cell make them ideal candidates for the study of mechanical properties. Note also that the values of ζ in Table 1 are for planar faces. Allowing the faces to warp can reduce the surface area even a little further as shown by the values for the Kelvin cell and the Weaire–Phelan cell, taken from Weaire and Phelan (1994).

Table 2
2-D surface properties

Cell	Sides (Average)	$\frac{L}{A^{2/3}}$
Triangular	3	$\frac{6}{(3)^{1/4}} = 4.56$
Square	4	4
Hex + Triang	4	$3(3)^{1/4} = 3.95$
Hexagonal	6	$\frac{2\sqrt{6}}{(3)^{1/4}} = 3.72$
Star (Star + Hex)	8	$\frac{6}{(3)^{1/4}} = 4.56$

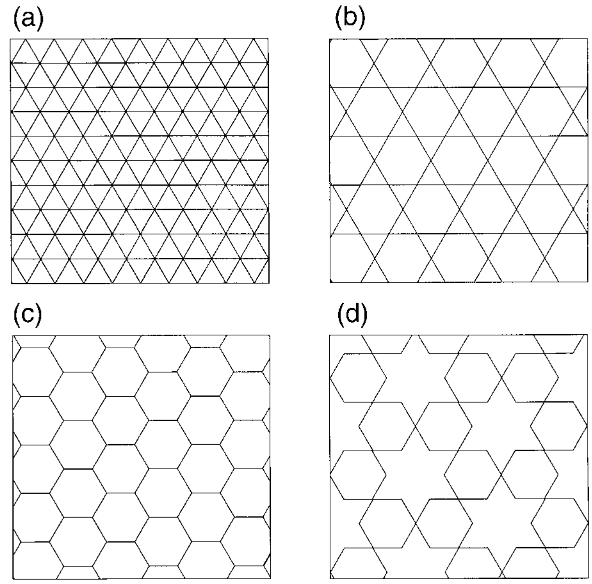

Fig. 1. 2-D Cell types: (a) Triang, (b) Hex and Triang, (c) Hex, (d) Star and Hex.

The quest for minimal surface forms can go further. Polydisperse cell forms can be constructed and evaluated. The standard approach begins with either regular or random point placements and then involves construction of Voroni cells, Kraynik et al. (1998). The resulting forms can be taken yet one step further by then relaxing the geometry to obtain minimal surfaces using the surface evolver program of Brakke (1992). It is found that some polydisperse cell forms can have even slightly lower surface content than the forms in Table 1.

Thus far, the three dimensional case has been discussed. The corresponding two dimensional case is equally well posed, although historically the significant interest in the three dimensional case came first. Table 2 shows the two dimensional cellular forms with the surface measure (surface per unit thickness) given by

$$\zeta_{2D} = \frac{L}{A^{1/2}},$$

where L is the cell perimeter and A its area. The cell types are shown in Fig. 1. In the cases where two cell forms are combined, the average ζ is found by averaging on ζ^2 with area weighting. All cases in Table 2 except the square cell have hexagonal symmetry. These hexagonal symmetry forms in Table 2 and Fig. 1 beginning with the triangular cell are the sequence of forms having hexagonal symmetry and maintaining the basic 60°, 120° angular architecture. The hexagonal cell has the minimal surface content, as is well understood and propagated as the ubiquitous 'honeycomb'. Many other two dimensional cellular forms could be considered, even non-periodic but repeating forms such as the Penrose pattern, Penrose (1979). Mathematicians have classified virtually all two dimensional patterns under the heading of patterns and tillings, Grünbaum and Shephard (1989).

The required local minimal characteristics in two and three dimensions are well understood. In 2-D, three cell sides must meet at 120° angles. In 3-D, four cell edges meet at tetrahedral angles, and three surfaces intersect at 120° angles to form the edges. A mathematically related minimal length (not surface) problem has been formulated by Christensen (1996).

Table 3
2-D mechanical properties

Microstructure	$\frac{\mu}{E_m}$	$\frac{K}{E_m}$	$\frac{E}{E_m}$	ν
Triangular	$\frac{1}{8}(1-c)$	$\frac{1}{4}(1-c)$	$\frac{1}{3}(1-c)$	$\frac{1}{3}$
Hexagonal	$\frac{3}{8}(1-c)^3$	$\frac{1}{4}(1-c)$	$\frac{3}{2}(1-c)^3$	1
Star Shaped	$\frac{3}{16}(1-c)^3$	$\frac{9}{16}(1-c)^3$	$\frac{9}{16}(1-c)^3$	1/2
GSCM	$\frac{1}{16}(1-c)^3$	$\frac{1}{4}(1-c)$	$\frac{1}{4}(1-c)^3$	1
Holes, Triang Pattern $c_M = \frac{\pi}{3\sqrt{3}}$	$\frac{1}{12c_M}(1-\frac{c}{c_M})^{\frac{1}{2}}$	$\frac{1}{6c_M}(1-\frac{c}{c_M})^{\frac{1}{2}}$	$\frac{2}{9c_M}(1-\frac{c}{c_M})^{\frac{1}{2}}$	1/3
Holes, Hex Pattern $c_M = \frac{\pi}{2\sqrt{3}}$	$\frac{1}{9c_M}(1-\frac{c}{c_M})^{\frac{3}{2}}$	$\frac{1}{12c_M}(1-\frac{c}{c_M})^{\frac{1}{2}}$	$\frac{4}{9c_M}(1-\frac{c}{c_M})^{\frac{3}{2}}$	1

These minimal surface forms are of great relevance to the mechanical performance problem because in many cases the methods of synthesis and manufacture are controlled by phase separation processes with surface tension being the controlling physical effect. Blown foams are the perfect example of this effect. The fundamental question to be answered is this. How do mechanical properties relate to these minimal surface forms, and how would the mechanical properties be effected by cellular forms that are far removed from the minimal surface forms?

It is advantageous to begin the examination of mechanical behavior with the two dimensional case because the path to understanding is less complicated in 2-D than in 3-D but still the 2-D case is of great interest and importance.

3. 2-D mechanical behavior

It is appropriate to begin with the 2-D microstructures that are shown in Fig. 1 and Table 2. Two-dimensional properties refer to states of plane stress or plane strain. Much of the concern here will be with the case of 2-D isotropy since the state of isotropy is the backbone for understanding all types of behavior. Isotropy of the mechanical properties is assured by hexagonal symmetry. Much of the work in this area was initiated by Gibson et al. (1982). The subject is broadly covered by the recent book of Gibson and Ashby (1997).

Table 3 is comprised of a modest compendium of elastic moduli for different microstructures in the range where the volume fraction of the voids are almost at the limit, beyond which material collapse occurs. Properties μ, K, E and υ are the effective 2-D plane stress shear modulus, bulk modulus, Young's modulus, and Poisson's ratio respectively. The quantity $(1-c)$ is the volume fraction of material, which is taken to be very small compared with one, for $c \to 1$. The term relative density is often used, where

$$\frac{\rho}{\rho_m} = (1-c),$$

with ρ being the density of the low density material form and ρ_m being the density of the composing material. The hexagonal case in Table 3 was first outlined by Gibson and Ashby. The triangular and star microstructure properties were given by Christensen (1995). A great variety of behaviors are evidenced in Table 3. Properties proportional to $(1-c)$ are recognized to be due to the direct extension or contraction of the material members, while properties proportional to $(1-c)^3$, at orders of magnitude lower levels, are due to bending of the material members. The triangular, hexagonal and star microstructure forms are probably the three classical forms delineating a variety of properties, certainly the first two are so standard as to be classical, and the third may complete the varied picture of different types of behavior. The triangular microstructure is the case of the civil engineering truss structure, well understood and practiced for a multitude of centuries. Probably the properties forms in Table 3 for the triangular case have been known for a very long time, but the original credit for these explicit formulas is unknown at this time. They at least go back to early modeling efforts with cellulose. The GSCM case is that of the Generalized Self Consistent Method, Christensen (1993), involving the packing of circular forms of a distribution of sizes. Some of these same cases as well as others are analyzed and discussed by Torquato et al. (1998).

The last two entries in Table 3 relate to cases where single size circular holes are placed in the 2-D medium. The holes are of two patterns, hexagonal packing wherein each hole has 6 nearest neighbors, and triangular packing where there are 3 nearest neighbors. The maximum packing fraction is given by c_{M} and the results shown from Day et al. (1992) are as $c \rightarrow c_{\mathrm{M}}$. Again a variety of behaviors are seen.

Considering the microstructures in Fig. 1, that of Fig. 1(b) the hex plus triang cell case, has properties the same as those of the triangular case in Table 3. The other three cases in Fig. 1 correspond to the first three entries in Table 3. A varied geometrical feature is involved in the star case of Fig. 1(d) compared with the other cases. From the interior of the hexagonal cells the geometry is seen to be concave, but for the star cell portion, some parts have outwardly convex regions. Such features are referred to as re-entrant angles. It is often said that cell geometries with re-entrant angles give rise to negative Poisson's ratios. However, the star cell case of Table 3 proves that this characteristic is not necessarily so. Actually it is in case of quite extreme anisotropy where such features as negative Poisson's ratios usually arise, as is discussed next.

There is an unlimited number of material arrangements that give isotropic behavior. When one broadens beyond that to consider anisotropy the possibilities become even broader in some sense. The consideration of anisotropic forms is now beginning to be considered in a manner that leads to applications. This is well illustrated by the work of Overacker et al. (1998). These authors have examined highly anisotropic 6 sided cellular forms involving re-entrant angles. They show that the negative Poisson's ratio effect itself is highly anisotropic, with negative values only occurring with respect to certain directions of loading relative to the microstructure. They discuss applications of such forms using the 'anchoring' effect related to negative Poisson's ratios. Further aspects of low density materials with negative Poisson's ratios will be given in the section on three dimensional microstructures and behaviors.

Consideration of strength is equally important as that of the stiffness and compliance properties. In the very low density region, the buckling of the micro-material members is the dominant mechanism. This occurs not only in a state of compression, but can occur in tension also. In this latter case the Poisson's ratio effect can induce buckling in members with an orientation transverse to the direction of loading. With regard to periodic microstructures, the strength properties can be highly anisotropic, even when the symmetry assures isotropy of the elastic moduli. Failure criteria in full, general tensorial form for these materials is not available, and remains to be developed. Specific discussions of failure mechanisms and behavior are given by Gibson and Ashby (1997).

The physical reality of most cellular forms is that they seldom conform to the idealized microstructure, due to manufacturing methods and tolerances and due to general aspects of usage. The degrading influences include imperfect connectedness and non-aligned micro-member geometry. Grenestedt (1998) has performed a study of the effect of the 'waviness'. It was shown that such imperfections, within the realistic range, have a strongly degrading effect upon properties. The implications of this are equally important to the three dimensional case.

A considerable degree of predictive order can be discerned in the microstructure types that have been studied and evaluated. The most important discrimination is that of the direct mechanism versus the bending mechanism of the internal loading of the material members. The minimal surface form, hexagonal, has a hybrid behavior with the 2-D bulk modulus controlled by the direct mechanism but the shear modulus controlled by the bending mechanism, see Table 3. The triangular cell form is totally controlled by the direct mechanism and the star cell form is totally controlled by the bending mechanism. Some simple rules of physical behavior are in evidence in these cases. The direct mechanism results when 6 material members meet at a node. The hybrid case occurs with 3 material members meeting at a node and finally the bending case occurs when a significant portion of the nodes have only two material members meeting, which is an idealization of a curved member. The microstructure of the hybrid case is the minimal surface form while the other two cases have microstructures far removed from the minimal surface form. All three cases have distinct and beneficial applications, as will be seen in the next section.

4. 2-D patterns with 3-D behavior

Most two dimensional forms when brought to the points of application become three dimensional cases. For example, the 2-D examples in Table 3 when viewed as 3-D continua are actually transversely isotropic materials. The properties in Table 3 are the in-plane properties which normally would be converted from plane stress to plane strain forms and must be supplemented by 3 more out of plane properties to give a complete characterization. Take axis 1 to be the symmetry axis. Then in the low density range, the first four microstructures in Table 3 all have the same out of plane properties given by

$$\frac{\mu_{12}}{\mu_{\mathrm{m}}} = \frac{1}{2}(1 - c),$$

$$\frac{E_{11}}{E_{\mathrm{m}}} = (1 - c)$$

and

$$\nu_{12} = \nu_{\mathrm{m}}.$$

The most important example of this class of materials is that of the honeycomb core for sandwich materials. As shown by Allen (1969), Noor et al. (1996) and Vinson (1999) the core in sandwich construction can be the limiting factor for this type of design. There are many other examples also, such as catalyst support in chemical exchange processes, crash protection materials, space filling forms, etc.

The in-plane strength and failure mechanisms of honeycomb materials have been extensively studied by Papka and Kyriakides (1998a, 1998b, 1998c). In general, instabilities develop locally and spread with increased loading. The stress strain curve typically exhibits a plateau region. These same authors also have studied cellular forms involving single size circular rings or cylinders packed in a hexagonal manner. The same general features of behavior evolve in this case. The out of plane compression of these types of materials generally involves a type of buckling followed by plastic deformation. Large amounts of energy can be absorbed during crushing, making such forms ideal for crash protection. This important topic is sufficiently self contained as to warrant separate treatment, Santosa and Wierzbicki (1998).

Returning to sandwich forms, there is an opportunity to develop different core material forms for various types of applications. For example, honeycomb does not easily conform to curved shapes, other types of cores are more suitable for this application.

5. 3-D mechanical behavior

The full blown case of low density materials in 3-D forms and applications is a rich, active and diverse field. In contrast to the 2-D case, a major new distinction arises in the 3-D case. This is the circumstance of the open cell form versus the closed cell form. Both cases are of great importance and must be treated separately. At first consideration it can be said that the open cell form favors strength, and the closed cell form favors stiffness, as idealizations, but the problem is much more subtle than that. Much depends on the technique and quality of manufacture, and often other design requirements take precedence, such as using closed cell forms to prevent moisture penetration.

First, consider the open cell case. The logical cell type to consider would be that of the Kelvin cell, the truncated octahedron. This has been done by Warren and Kraynik (1997) and by Zhu et al. (1997a). Several new ingredients arise in the 3-D case that don't exist in the 2-D case. First, for those materials that are synthesized by a phase separation process, there is a process sequence of forming the minimal surface cell followed by evacuation of the material from the faces into an aggregation along the cell edges, leaving the open cell form. The consequence of this process is that the cross section of the material members is formed by the intersection of three Plateau borders, Kraynik et al. (1998), involving convex circular arcs or some approximation thereof. Another complication is that the micro-material members can not only undergo bending as in the 2-D case, but can undergo torsion as well. It can be shown that the torsion effects are of the same order as the bending effects and cannot be neglected.

Despite these complications, remarkable new results have been obtained in the references cited above. The Kelvin cell aggregation has cubic symmetry, with 3 independent elastic properties. Warren and Kraynik (1997) and Zhu et al. (1997a) show that the properties are almost isotropic, to within a few percent. In one of the most important results of recent times, Warren and Kraynik (1997) reveal the special case in which the cubic properties form reduces exactly and identically to the case of isotropy. This case is that which occurs when the material members have a circular cross section and when the material Poisson's ratio is $\nu_m = 0$. In this case, their results give the isotropic bulk modulus and shear modulus as

$$\frac{k}{E_m} = \frac{1}{9}(1-c)$$

and

$$\frac{\mu}{E_m} = \frac{4\sqrt{2}}{9\pi}(1-c)^2 = 0.200070(1-c)^2 \cong \frac{1}{5}(1-c)^2.$$

The fact that this case involves circular cross section members, rather than those with Plateau borders does not detract from its significance. The circular case can be taken as the base line behavior. The shear properties for the Plateau borders case are considerably larger than the above value due to the increased bending and torsional rigidity. Kraynik et al. (1998) discusses results for other cell types, such as the Weaire–Phelan cell and poly-disperse forms.

It is seen that the properties just stated involve the bulk modulus dominated by the direct mechanism of deformation while the shear modulus is given by the bending (and torsion) mechanism. This situation is akin to that which occurred in the 2-D case, both cases with cell forms simulating the minimal surface cell.

As with the 2-D case, there is a behavior that involves only the direct mechanism. This isotropic result was first given by Gent and Thomas (1959), as deduced generally, not from any particular cell type. The bulk modulus is the same as given above, but the shear modulus and Young's modulus are given by

$$\frac{\mu}{E_{\mathrm{m}}} = \frac{1}{15}(1-c)$$

and

$$\frac{E}{E_{\mathrm{m}}} = \frac{1}{6}(1-c).$$

It should also be noted that Ko (1965) was one of the very early contributors to the field with studies of several cell types. Gibson and Ashby (1982) analyze and discuss 3-D material behavior in terms of all properties involving the bending mechanism.

Now consider closed cell forms. In recent work Christensen (1998) has used the Generalized Self Consistent Method to obtain the following predictions for the isotropic bulk and shear moduli

$$\frac{k}{E_{\mathrm{m}}} = \frac{2}{9(1-\nu_{\mathrm{m}})}(1-c)$$

and

$$\frac{\mu}{\mu_{\mathrm{m}}} = \frac{1}{3}(1-c).$$

Kraynik et al. (1998) have used and discussed finite element models to obtain numerical results for the case of $\nu_{\mathrm{m}}=0.49$. These results for the Kelvin cell and the Weaire–Phelan cell are nearly isotropic and in quite close agreement with the above analytical solution results. The shear moduli involve a direct resistance mechanism as shown by the $(1-c)$ dependence in contrast to the open cell case. It is quite interesting to note that the above analytical solution predicts the value for μ/μ_{m} to be independent of Poisson's ratio ν_{m}. This unexpected result could be checked against numerical solutions. It is seen that the closed cell form, from the point of view of effective moduli, provides a much more efficient material utilization than does the open cell form.

The large deformation properties of low density materials presents a challenge. Budiansky and Kimmel (1987) considered forms appropriate for the behavior of lung tissue. Zhu et al. (1997b) have studied open cell forms using the Kelvin cell and performing elastic type analyses with local material buckling at large compressive strains. They find a strong dependence of the nonlinear stress strain curve upon the direction of deformation. Even though the form is nearly isotropic in the small strain range, there is no reason to expect any form of isotropy in the large deformation range. Although typical stress strain curves often show a plateau type behavior at sufficiently large strains, Kraynik et al. (1998), Zhu et al. (1997b) find some cases in which no plateau region exists. Strength considerations are not extensively treated for these materials, and this represents a major opportunity area. Compressive strength is the limiting condition due to buckling of the micro-material members, Gibson and Ashby (1997). At the same volume fraction, the characteristic material thickness will be much less in the closed cell form than in the open cell form, and this favors the open cell form for strength. General considerations on the effect of imperfections, Grenestedt (1998), are probably even more important in the three dimensional case than in 2-D.

Low density material forms, both closed cell and open cell, are made with the full spectrum of material types, metals, polymers and ceramics. Polymers have been the traditional materials for these forms, but the other materials are also rapidly progressing. A survey of opportunities with cellular metallic forms is given by Evans et al. (1998a). Related studies are by Evans et al. (1998b) and Sugimura et al. (1998). Nieh et al. (1998) have given detailed morphology characterizations using synchrotron X-ray tomography for aluminum open cell forms and shown the mechanical behavior to be in accordance with the theoretical forms in the literature, and surveyed here. General aspects of the subject have been discussed by Koch et al. (1994). It is likely that a very wide variety of forms and applications will emerge using all material types in cellular form.

As mentioned in connection with two dimensional behavior, negative Poisson's ratio effects can be induced by controlling the material microstructure. Choi and Lakes (1992, 1995) have shown how re-entrant angle forms can produce these effects and how the effect can be employed. This is in fact a good example of how cellular materials can be 'designed' to produce certain desired effects. Sigmund (1995) has developed a methodology for designing materials in this sense. Other general aspects of cellular, porous materials are given by Fleck and Cocks (1997) and an earlier text by Hilyard (1982).

Up until this point, all low density material types have been based upon specific cellular forms. This characteristic is not requisite, low density forms can exist with no discernible cellular morphology. Aerogel materials are the perfect examples of such forms. These materials exist in the ultra low density range and morphological characteristics are best described on a scale not too much larger than that of angstroms, with forms sometimes called a 'string of pearls' type network. Mechanical characterization of these materials have been given by Scherer et al. (1995a, 1995b), as well as Pirard and Pirard (1997). A particular case of the quadratic dependence of modulus upon $(1-c)$ is given by Bellet et al. (1996). A general mechanical summary was that of LeMay et al. (1990), and recent considerations of applications by Hrubesh (1998).

With regard to sandwich type applications for closed cell forms in the cores, Hutchinson and He (1998) have given the relevant structural analysis, while Budiansky (1999) considers minimum weight design for structures, which certainly favors sandwich construction. Conference proceedings on sandwich construction are given as Fourth International Conference on Sandwich Construction (1998). A trade magazine with current trends is that of the Professional Boatbuilder, 1998.

Any summary of low density material, cellular forms would not be complete without mentioning the mechanics of bio-materials in general and of bone in particular. However, that topic is too extensive and too important to merely be mentioned here in summary reference. Entry to this field can be gained from Kinney and Ladd (1998), Yang et al. (1998) and Ladd and Kinney (1997).

6. Future directions

There are many opportunities for new applications of light weight materials, and ultra light weight structural forms. Such materials will be tailored to specific applications. Understanding and controlling the degree of anisotropy in both 2-D and 3-D applications will need to receive much attention. In a more general sense, controlling the microstructure to obtain desired properties will be of great importance. This ties in with the methods of material synthesis and production. Understanding and controlling non-ideal morphological geometry and imperfections will need to receive much more attention, particularly for materials in the very low density range. Ultimately it will probably be possible to design materials or at least design material microstructures for each and every application. The economics of production and life cycle cost will be one of the first order requirements.

In terms of specific mechanics of materials directions, understanding the direct mechanism versus the bending (and torsion) mechanism will be of importance for 3-D open cell forms. Until the 3-D case is understood with the same degree of rigor that exists in the 2-D case there will be missing information.

For example, what explicit 3-D microstructure(s) will give the direct mechanism for all properties. Perhaps the first entry in Table 1 is the guiding form for the direct mechanism. It would involve the intersection of 12 material members at each joint. Such a microstructure would be far removed from the minimal surface cell form. Explicit 3-D microstructures for all properties being controlled by the bending and torsion mechanism are equally obscure at this point. Strength characterizations will be necessary for design applications. Tensorial forms for strength criteria will be needed to account for multi-component stress states.

Sandwich forms will be of growing importance and dominance. Such forms are already an integral part of the aerospace industry. As costs are driven down, sandwich forms based upon cellular cores (both 2-D patterns and 3-D forms) will come into widespread usage in marine/naval applications and ultimately in automobiles and trucks. The core is often the limiting factor in sandwich structures, and it merits a full scale attack on all related problems.

Acknowledgements

It is a pleasure to acknowledge the contribution of many others in preparing this article. Dr. Andrew Kraynik, who has performed much of the original work in this area, was extremely helpfull in assessing the status and relevance of many of the contributions. Professor George Scherer was of much help in considering aerogels. A large number of other people kindly supplied me with individual articles and positive suggestions.

This work was prepared with support from the Office of Naval Research, Dr. Y.D.S. Rajapakse, program monitor and under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

References

Allen, H.G., 1969. Analysis and Design of Sandwich Panels. Pergamon Press, Oxford.
Bellet, D., Lamagnére, P., Vincent, A., Brechet, Y., 1996. J. Appl. Phys. 80, 3772–3776.
Brakke, K.A., 1992. The surface evolver. Experimental Mathematics 1, 141–165.
Budiansky, B., Kimmel, E., 1987. Elastic moduli of lungs. J. Appl. Mech. 54, 351–358.
Budiansky, B., 1999. On the minimum weights of compression structures. Int. J. Solids, Structs. 36, 3677–3708.
Choi, J.B., Lakes, R.S., 1992. Nonlinear properties of metallic cellular materials with a negative Poisson's ratio foams. J. Mater. Sci. 27, 5375–5381.
Choi, J.B., Lakes, R.S., 1995. Analysis of elastic modulus of conventional foams and of re-entrant foam materials with a negative Poisson's ratio. Int. J. Mech. Sci. 37, 51–59.
Christensen, R.M., 1993. Effective properties of composite materials containing voids. Proc. Royal Soc. Lond. 440, 461–473.
Christensen, R.M., 1995. The hierarchy of microstructures for low density materials. Z. Angew. Math. Phys. 46, 5506–5521.
Christensen, R.M., 1996. On the relationship of minimal conditions to low density material microstructures. J. Mech. Phys. Solids 44, 2113–2123.
Christensen, R.M., 1998. Two theoretical elasticity micromechanics models. J. Elast. 50, 15–25.
Day, A.R., Snyder, K.A., Garboczi, E.J., Thorpe, M.F., 1992. The elastic moduli of a sheet containing circular holes. J. Mech. Phys. Solids 40, 1031–1051.
Evans, A.G., Hutchinson, J.W., Ashby, M.F., 1998a. Cellular Metals, MECH 326. Harvard University.
Evans, A.G., Hutchinson, J.W., Ashby, M.F., 1998b. The Thermostructural Performance and Multifunctionality of Cellular Metal Systems, MECH 323. Harvard University.
Fleck, N.A., Cocks, A.C.F. (Eds.), 1997. Mechanics of Granular and Porous Materials. Kluwer, Dordrecht.
Fourth International Conference on Sandwich Construction, 1998. Royal Inst. of Technology, Stockholm, Sweden.
Gent, A.N., Thomas, A.G., 1959. The deformation of foamed elastic materials. J. Appl. Polymer Sci. 1, 107–113.
Gibson, L.J., Ashby, M.F., 1982. The mechanics of three-dimensional cellular materials. Proc. Royal Soc. Lond. A-382, 43–59.
Gibson, L.J., Ashby, M.F., Schajer, G.S., Robertson, C.I., 1982. The mechanics of two-dimensional cellular materials. Proc. Royal Soc. Lond. A-382, 25–42.

Gibson, L.J., Ashby, M.F., 1997. Cellular Solids: Structure and Properties, 2nd Ed. Cambridge University Press, Cambridge.

Grenestedt, J.L., 1998. Influence of wavy imperfections in cell walls on elastic stiffness of cellular solids. J. Mech. Phys. Solids 46, 29–50.

Grünbaum, B., Shephard, G.C., 1989. Tilings and Patterns, An introduction. W. H. Freeman, New York.

Hilyard, N.C. (Ed.), 1982. Mechanics of Cellular Plastics. MacMillan, New York.

Hrubesh, L.W., 1998. Aerogel applications. J. Non-Crystalline Solids 225, 335–342.

Hutchinson, J.W., He, M.Y., 1998. Buckling of Cylindrical Sandwich Shells with Metal Foam Cores, MECH 332. Harvard University.

Lord Kelvin (Thompson W.), 1887. On the division of space with minimum partitional area. Philos. Mag. 24, 503–514.

Kinney, J.H., Ladd, A.J.C., 1998. The relationship between three-dimensional connectivity and the elastic properties of trabecular bone. J. Bone and Mineral Res. 13, 839–845.

Ko, W.L., 1965. Deformations of foamed elastomers. J. Cellular Plastics 1, 45–50.

Koch, U., Thompson, M.S., Nardone, V.C., 1994. Structure and properties of industrial aluminum foams. In: 4th Int. Conf. on Aluminum Alloys, vol. I. Georgia Inst. of Tech, Atlanta.

Kraynik, A.M., Neilsen, M.K., Reinelt, D.A., Warren, W.E., 1998. Foam micromechanics. In: Proc. NATO Advanced Study Institute on Foams, Emulsions and Cellular Materials. Kluwer.

Kraynik, A.M., Warren, W.E., 1994. The elastic behavior of low-density cellular plastics. In: Hilyard, N.C., Cunningham, A.C. (Eds.), Low Density Cellular Plastics. Chapman and Hall, London, pp. 187–225.

Ladd, A.J.C., Kinney, J.H., 1997. Elastic constants of cellular materials. Physica A-240, 349–360.

LeMay, J.D., Hopper, R.W., Hrubesh, L.W., Pekala, R.W., 1990. Low density microcellular materials. Mats. Res. Soc. Bull. 15, 19–45.

Nieh, T.G., Kinney, J.H., Wadsworth, J., 1998. Morphology and elastic properties of aluminum foams produced by a casting technique. Scripta Materialia 38, 1487–1494.

Noor, A.K., Burton, W.S., Bert, C.W., 1996. Computational models for sandwich panels and shells. Appl. Mech. Reviews 49, 155–199.

Overacker, D.W., Cuitiño, A.M., Langrana, N.A., 1998. Effects of morphology and orientation on the behavior of two-dimensional hexagonal foams and application in a re-entrant foam anchor model. Mech. Mats. 29, 43–52.

Papka, S.D., Kyriakides, S., 1998a. Experiments and full-scale numerical simulations of in-plane crushing of a honeycomb. Acta. Mater. 46, 2765–2776.

Papka, S.D., Kyriakides, S., 1998b. In-Plane Biaxial Crushing of Honeycombs, Part I: Experiments. University of Texas, Austin, MSS&M Report No. 98/1.

Papka, S.D., Kyriakides, S., 1998c. In-Plane Biaxial Crushing of Honeycombs, Part II: Analysis. University of Texas, Austin, MSS&M Report No. 98/2.

Penrose, R., 1979. A class of non-periodic tilings of the plane. Math. Intelligencer 2, 32–37.

Plateau, J.A.F., 1873. Statique Experimentale et Theorique des Liquides Soumis aux Seules Forces Moleculaires. Gauthier–Villiard.

Pirard, R., Pirard, J.-P., 1997. Aerogel compression theoretical analysis. J. Non-Crystalline Solids 212, 262–267.

Professional Boatbuilder, 1998. Child Series.

Santosa, S., Wierzbicki, T., 1998. On the modeling of crush behavior of a closed-cell aluminum foam structure. J. Mech. Phys. Solids 46, 645–669.

Scherer, G.W., Smith, D.M., Stein, D., 1995a. Deformation of aerogels during characterization. J. Non-Crystalline Solids 186, 309–315.

Scherer, G.W., Smith, D.M., Qui, X., Anderson, J.M., 1995b. Compression of aerogels. J. Non-Crystalline Solids 186, 316–320.

Sigmund, O., 1995. Tailoring materials with prescribed elastic properties. Mech. Mats. 20, 351–368.

Sugimura, Y., Rabiei, A., Evans, A.G., Harte, A.M., Fleck, N.A., 1998. Compression Fatigue of a Cellular Alloy, MECH 331. Harvard University.

Torquato, S., Gibiansky, L.V., Silva, M.J., Gibson, L.J., 1998. Effective mechanical and transport properties of cellular solids. Int. J. Mech. Sci. 40, 71–82.

Vinson, J.R., 1999. The Behavior of Sandwich Structures of Isotropic and Composite Materials. Technomic Pub. Co, Lancaster, PA.

Warren, W.E., Kryanik, A.M., 1997. Linear elastic behavior of a low-density Kelvin foam with open cells. J. Appl. Mech. 64, 787–794.

Weaire, D., Phelan, R., 1994. A counter-example to Kelvin's conjecture on minimal surfaces. Phil. Mag. Lett. 69, 107–110.

Yang, G., Kabel, J., van Rietbergen, B., Odgaard, A., Huiskes, R., Cowin, S.C., 1998. A New Method for the Analysis of the Dependence of the Elastic Constants of Cancellous Bone upon Volume Fraction. Dept. Mech. Eng., The City University of New York.

Zhu, H.X., Knott, J.F., Mills, N.J., 1997a. Analysis of the elastic properties of open-cell foams with tetrakaidecahedral cells. J. Mech. Phys. Solids 45, 319–343.

Zhu, H.X., Mills, N.J., Knott, J.F., 1997b. Analysis of the high strain compression of open-cell foams. J. Mech. Phys. Solids 45, 1875–1904.